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Common funds investment portfolio optimization with fuzzy approach

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Abstract

Conventional portfolio optimization models assume that future of the Stock Market will be predicted by past data. However, regardless of whether how accurate is the past data, this theorem in financial market is not applicable due to the high volatility of the market environment. This research is about optimization problem of fuzzy set that shows the assets return by fuzzy data. Part of the data of the actual financial information, are information from the actual data of Years 2012 and 2013 that have been obtained as fragile (and final) and another part of the survey experts as predictive information was obtained for the years 2014 to 2017 in the form of triangular fuzzy numbers. To optimize portfolio, nonlinear mathematical models for some were specified and presented then using the change of variables technique that in operations research literature is a simple technique, two models could merged and integer linear model variables were created and the results were used to calculate the software Lingo. Finally the results were obtained in accordance with a basics idea of financial economics that whatever the degree of investment risk, is more he want to get more return.

Keywords: Return; risk; common investment fund; triangular fuzzy number; fuzzy model; fuzzy data

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1. Introduction

Financial problems in recent years, has been the subject of many investigations. Portfolio theory states that: everyone assets and investments should be portfolio (diversification).Namely instead of investing in an asset, portfolio risk can be minimized with portfolio investment. So portfolio selection is a vital activity in all organizations for complex processes in the different case and sometimes contradictory. (Lin &Hesy 2004) Among financial

Scenarios investment funds is proposed as one of the most popular financial strategies. (Chang 2010) investment funds are financial intermediaries that collect the financial resources for individuals and companies and investing in diversified portfolio of securities. Although joint investment fund industry in Iran begins with a multi-ten-year delay but were established with warm reception from investors in 2007. (Roshangarzadeh Amin, Ahmadi, Mohammad Ramadan). The main goal of investors is gaining more return on acceptable risk level. Accordingly, whatever Investor can predict better increased the future returns. Forecasting future returns of mutual funds has particular importance for Investors. (Astronomer, 2009).Fund portfolio optimization is very important point, In addition to investing in the Fund. So that to have the greatest benefit with the least possible risk for Investors. The Majority of studies that have been done so far in the portfolio selection is based on Markowitz approach and He proposed a mathematical model which is based on the mean-variance. Due to the lack of adequate mathematical models of hard or firm, on the cover of uncertainty, Complexity and ambiguity or concepts and inaccurate variables, Using the principles and methods of fuzzy becomes necessary. (Elton &Gaber 1995) in a real environment, Unreliability in the trends in market and inability to meet it, can be followed by very detrimental effects. To address some vague information in Zadeh decision-making, (Zadeh 1965) introduced the concept phase.

2. Conceptual Model

Assume that we have n property for Investment and M 0 is a Part of the available total budget that should be in each box to be invested and j=1,2,…,n،X_j represents the invested dollar amount that is in j fund. table 1 illustrates the limitation and equation 1 to 31 prove the conceptual model.

Table 1. Our Limitation

The introduction of variable	Variable name
Random variable that Shows the rate of return on j asset	R_j
The high level of investment on j asset	U_j
Expected return j asset	r_j
Total funds available	M_0
Return	R_0
The ratio of total capital that Invested in j	X_j
Have Implying the realization of a R j random variable during the t course	r_{jt}

$$x_j \geq 0 \,, \sum_{j=1}^{n} x_j = M. \tag{1}$$

Rj Random variable that Shows the rate of return on j asset.The expected return on assetsis displayedin this case:

$$r(x_1, \dots, x_n) = \mathrm{E}[\sum_{j=1}^{n} R_j X_j] = \sum_{j=1}^{n} [R_j] X_j \tag{2}$$

Used to measure the potential risk:

$$\delta(x_1, \dots, x_n) = \sqrt{\mathrm{E}[\sum_{j=1}^{n} R_j X_j] = \sum_{j=1}^{n} [R_j] X_j\}^2]} \tag{3}$$

Portfolio problem is formulated as a quadratic programming problem:

$$V=\text{Min}\sum_{i=1}^{n}\sum_{j=1}^{n}\delta_{ij}X_iX_j \quad (4)$$

$$\text{s.t} \quad \sum_{j=1}^{n}X_j = M_. \quad (5)$$

$$\sum_{j=1}^{n}r_jX_j \geq R_0 \quad (6)$$

$$0\leq X_j \leq U_j, j=1,\dots,n \quad (7)$$

Where R 0 is the return of the dollar, $r_J$and U $_J$ is expected return and high levels of investment in j asset and δ_{ij} is standard deviation of returns.

$$V=\text{Min } E\left[\left|\sum_{j=1}^{n}R_jX_j - E\left[\sum_{j=1}^{n}R_jX_j\right]\right|\right] \quad (8)$$

$$\text{s.t. } \sum_{j=1}^{n}X_j = M_0 \quad (9)$$

$$\sum_{j=1}^{n}E[R_j]X_j \geq R_0 \quad (10)$$

$$00 \leq x_j \leq U_j, j=1,\dots,n \quad (11)$$

Suppose historical data assets for the previous T year, Price volatility and dividend payments were given. Then we can estimate the investment returns of any assets from the past data. rjt Implies the realization of the R j random variable during the T course.

$$r_j = E\left[R_j\right] = \frac{1}{T}\sum_{t=1}^{T}r_{jt} \quad (12)$$

$$V=\text{Min } \sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt}-r_j)X_j\right|/T \quad (13)$$

$$s.t. \quad \sum_{j=1}^{n}X_j = M_0 \quad (14)$$

$$\sum_{j=1}^{n} r_j X_j \geq R_0 \tag{15}$$

$$0 \leq X_j \leq U_j, \quad \text{j=1,…,n} \tag{16}$$

This model can be transformed into the following linear program:

$$V = Min \sum_{t=1}^{T} \frac{U_t}{T} \tag{17}$$

$$\text{s.t.} \quad U_t + \sum_{j=1}^{n} (r_{jt} - r_j) X_j \geq 0, \quad \text{t=1,…,T} \tag{18}$$

$$U_t - \sum_{j=1}^{n} (r_{jt} - r_j) X_j \geq 0, \quad \text{t=1,…,T} \tag{19}$$

$$\sum_{j=1}^{n} X_j = M_0 \tag{20}$$

$$\sum_{j=1}^{n} r_j X_j \geq R_0 \tag{21}$$

$$0 \leq X_j \leq U_j, \quad \text{j=1,…,n} \tag{22}$$

To find the value of fuzzy goal, finding up and down function limit is enough. We also have:

$$V_{\propto}^{u} = Max_X \text{V=} \sum_{t=1}^{T} \frac{u_t}{T} \tag{23}$$

$$\text{s.t.} \quad u_t + \sum_{j=1}^{n} r_{jt} X_j - \sum_{j=1}^{n} r_j X_j \geq 0\,, \quad \text{t=1,…,T} \tag{24}$$

$$u_t - \sum_{j=1}^{n} r_{jt} X_j - \sum_{j=1}^{n} r_j X_j \geq 0\,, \quad \text{t=1,…,T} \tag{25}$$

$$\sum_{j=1}^{n} X_j = M_0 \tag{26}$$

$$\sum_{j=1}^{n} r_j X_j \geq R_0 \tag{27}$$

$$\left(R_{jt}\right)_{\propto}^{l} \le r_{jt} \le \left(R_{jt}\right)_{\propto}^{u} \quad \text{,j=1,…,n , t=1,…,T} \tag{28}$$

$$\sum_{t=1}^{T} \frac{\left(R_{jt}\right)_{\propto}^{l}}{T} \le r_j \le \sum_{t=1}^{T} \frac{\left(R_{jt}\right)_{\propto}^{u}}{T} \tag{29}$$

$$0 \le X_j \le U_j, \quad \text{j=1,…,n} \tag{30}$$

By changing the variables and solving the dual issue, we can find a reply for the primary issue $\boldsymbol{X_j^*}$,j=1,…,n The portfolio returns are calculated in this case:

$$R_{\propto}^{u} = \sum_{j=1}^{n} r_j^* X_j^* = \frac{\sum_{j=1}^{n}\sum_{t=1}^{T} r_{jt}^* X_j^*}{T} \tag{31}$$

Numerical solutions for $\boldsymbol{V_{\propto}^{u}}$, $\boldsymbol{V_{\propto}^{l}}$ is possible in α different levels. The approximation figures for L(v) and R(v) were Obtained. Also the membership function of portfolio returns $\boldsymbol{\tilde{R}}$ were calculated.

Upper bound of the objective function $\boldsymbol{V_{\propto}^{u}}$ and the lower limit of the objective function $\boldsymbol{V_{\propto}^{l}}$ were obtained at a certain level of cutα.

3. Research findings

To collect numerical and verbal data five boxes in stock was selected randomly and asked from about 30 financial experts and activists stock market forecast of future returns (4 future years) listed each fund to three number:

- Prime number : the lowest predictable return for selected fund in the target year
- The second number : Average predictable return for selected fund in the target year
- The third number : maximum predictable return for selected fund in the target year

Arithmetic mean of these numbers (In the form of triangular fuzzy number) was calculated as Fund future returns in the target year. For each fund 365 last days return and 6 last month and (Maximum, minimum and average weighted) annual return from date of establishment as yet also Fund return from date of establishment as mentioned For more information of forecasters.

4. Results of the gathering expert opinions and actual data

As mentioned in the previous chapter, Information obtained from actual data from the Years 2012 and 2013 that have been obtained as fragile (and definitive) must be gathered to information on predicted fuzzy years 2014 to 2017 together. Therefore, the results of these actions are determined and illustrated in table 2.

Table 2.The results of the gathering expert opinions and actual data

	Fund Name				
Return year	Informed X_1	Brokerage BSI X_2	Insurance Exchange X_3	Broker Keeper X_4	National Bank X_5

2013	0.476	0.005	0.272	0.210	0.520
Forecast2014	(0.086 ,0.176 ,0.274)	(0.037 ,0.158 ,0.252)	(0.053 ,0.151 ,0.233)	(0.070 ,0.188 ,0.285)	(0.067 ,0.153 ,0.256)
Forecast2015	(0.183 ,0.297 ,0.447)	(0.131,0.249 ,0.377)	(0.141 ,0.244 ,0.344)	(0.173 ,0.285 ,0.441)	(0.163 ,0.269 ,0.386)
Forecast2016	(0.289 ,0.417 ,0.575)	(0.201 ,0.350 ,0.494)	(0.206 ,0.326 , 0.472)	(0.259 ,0.386 ,0.526)	(0.243 ,0.348 , 0.488)
Forecast2017	(0.306 ,0.442 ,0.631)	(0.307 ,0.436 ,0.617)	(0.268 ,0.385 ,0.551)	(0.320 ,0.435 ,0.611)	(0.298 ,0.414 ,0.611)
Expected	(0.245 ,0.323 ,0.422)	(0.187 ,0.273 ,0.364)	(0.169,0.242,0.324)	(0.184,0.263,0.358)	(0.169,0.242,0.324)

It is necessary to be determined as well some of the most important parameters in this model until to be able to solve the model. Therefore, based on the fact that it has faced in the Portfolio selection, proposed the following parameters:

n this basis, the following general restrictions would be contrary to the description. The maximum available budget for example $M_0 = 200$ Million Tomans $\sum_j x_j = M_0$ and the maximum budget for each fund is 90 million Tomans.

$$0 \leq x_1, x_2, x_3, x_4, x_5 \leq U_j = 90$$

The annual returns value of 20٪ namely$(R_0 = 20\%)$ $M_0 + 20\%M_0 = 240$ will be determined.

$$(0.245,0.323,0.422)X_1 + (0.187,0.273,0.364)X_2 + (0.169,0.242,0.324)X_3 + (0.184,0.263,0.358)X_4 + (0.285,0.354,0.447)X_5 \geq 240$$

5. Providing the final model with case study numbers

The final model can be presented as equations 32-47

$$\bar{V} = \frac{\min\{u_1,u_2,u_3,u_4,u_5\}}{6} \tag{32}$$

s.t: (33)

$$\mathrm{u}_1 + [0/132\mathrm{x}_1 + 0/441\mathrm{x}_2 + 0/072\mathrm{x}_3 + 0/073\mathrm{x}_4 + 0/420\mathrm{x}_5 - (0/245,0/323,0/422)\mathrm{x}_1 - (0/187,0/273,0/364)\mathrm{x}_2 - (0/169,0/242,0/324)\mathrm{x}_3 - (0/184,0/263,0/358)\mathrm{x}_4 - (0/169,0/242,0/324)\mathrm{x}_5] \geq 0$$

$$\mathrm{u}_2 + [0/476\mathrm{x}_1 + 0/005\mathrm{x}_2 + 0/272\mathrm{x}_3 + 0/210\mathrm{x}_4 + 0/520\mathrm{x}_5 - (0/245,0/323,0/422)\mathrm{x}_1 - (0/187,0/273,0/364)\mathrm{x}_2 - (0/169,0/242,0/324)\mathrm{x}_3 - (0/184,0/263,0/358)\mathrm{x}_4 - (0/169,0/242,0/324)\mathrm{x\_5}] \geq 0 \tag{34}$$

$$\mathrm{u}_3 + [(0/086 ,0/176 ,0/274)\mathrm{x}_1 + (0/037 ,0/158 ,0/252)\mathrm{x}_2 + (0/053 ,0/151 ,0/233) + (0/070 ,0/188 ,0/285)\mathrm{x}_4 + (0/067 ,0/153 ,0/256)\mathrm{x}_5 - (0/245,0/323,0/422)\mathrm{x}_1 - (0/187,0/273,0/364)\mathrm{x}_2 - (0/169,0/242,0/324)\mathrm{x}_3 - (0/184,0/263,0/358)\mathrm{x}_4 - (0/169,0/242,0/324)\mathrm{x}_5] \geq 0 \tag{35}$$

$$u_4 + [(0/183\ ,0/297\ ,0/447\ ))x_1 + (\ 0/131,0/249\ ,0/377\ )x_2(0/141\ ,0/244\ ,0/344\ ) + (0/173\ ,0/285\ ,0/441\ )x_4 + (0/163\ ,0/269\ ,0/386\ )x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (36)$$

$$u_5 + [(0/289\ ,0/417\ ,0/575\ ))x_1 + (0/201\ ,0/350\ ,0/494\ )x_2 + (0/206\ ,0/326\ ,0/472) + (0/259\ ,0/386\ ,0/526\ )x_4 + (0/243\ ,0/348\ ,0/488)x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (37)$$

$$u_6 + [(0/306\ ,0/442\ ,0/631\ ))x_1 + (0/307\ ,0/436\ ,0/617\ )x_2 + (0/268\ ,0/385\ ,0/551\ ) + (0/320\ ,0/435\ ,0/611\ )x_4 + (0/298\ ,0/414\ ,0/611\ )x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (38)$$

$$u_1 - [0/132x_1 + 0/441x_2 + 0/072x_3 + 0/073x_4 + 0/420x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (39)$$

$$u_2 - [0/476x_1 + 0/005x_2 + 0/272x_3 + 0/210x_4 + 0/520x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (40)$$

$$u_3 - [(0/086\ ,0/176\ ,0/274\ )x_1 + (0/037\ ,0/158\ ,0/252\ )x_2 + (0/053\ ,0/151\ ,0/233\ ) + (0/070\ ,0/188\ ,0/285\ )x_4 + (0/067\ ,0/153\ ,0/256\ )x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0.169,0.242,0.324)x_5] \geq 0 \quad (41)$$

$$u_4 - [(0/183\ ,0/297\ ,0/447\ ))x_1 + (\ 0/131,0/249\ ,0/377\ )x_2(0/141\ ,0/244\ ,0/344\ ) + (0/173\ ,0/285\ ,0/441\ )x_4 + (0/163\ ,0/269\ ,0/386\ )x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (42)$$

$$u_5 - [(0/289\ ,0/417\ ,0/575\ ))x_1 + (0/201\ ,0/350\ ,0/494\ )x_2 + (0/206\ ,0/326\ ,0/472) + (0/259\ ,0/386\ ,0/526\ )x_4 + (0/243\ ,0/348\ ,0/488)x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (43)$$

$$u_6 - [(0/306\ ,0/442\ ,0/631\ ))x_1 + (0/307\ ,0/436\ ,0/617\ )x_2 + (0/268\ ,0/385\ ,0/551\ ) + (0/320\ ,0/435\ ,0/611\ )x_4 + (0/298\ ,0/414\ ,0/611\ )x_5 - (0/245,0/323,0/422)x_1 - (0/187,0/273,0/364)x_2 - (0/169,0/242,0/324)x_3 - (0/184,0/263,0/358)x_4 - (0/169,0/242,0/324)x_5] \geq 0 \quad (44)$$

$$\sum_{j=1/5} X_j = 200 \quad (45)$$

$$0 \leq \mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4, \mathrm{x}_5 \leq \mathrm{U_j} = 90 \quad (46)$$

$$(0.245,0.323,0.422)X_1 + (0.187,0.273,0.364)X_2 + (0.169,0.242,0.324)X_3 + (0.184,0.263,0.358)X_4 + (0.285,0.354,0.447)X_5 \geq 240 \quad (47)$$

Based on the above model and necessity for the implementation of various alpha cuts, the slightly model should be coding in Software Lingo Version 8.Lingo 8 alpha output to alpha zero is equal to the following values:

Also according to the needs to investigating, tables 3,4,5 are provided in following for one and half alpha.

Table 3.Results of the discussed variables in alpha 0.5

Amount	Variable
90	X_1
66	X_2
3	X_3
32	X_4
9	X_5
12.1	U_1
13.9	U_2
14.7	U_3
10.3	U_4
11.2	U_5
10.9	U_6

Table 4. Results of the discussed variables in Alpha 0.5

Amount	Variable
90	X_1
2	X_2
10	X_3
62	X_4
36	X_5
17.3	U_1
16.9	U_2
14.8	U_3
15.6	U_4
16.2	U_5
17.1	U_6

Table 5.Results of the discussed variables in Alpha 1

Amount	Variable
89	X_1
3	X_2
35	X_3
48	X_4
25	X_5
19.1	U_1
19.8	U_2
18.9	U_3
18.7	U_4
17.9	U_5
17.35	U_6

Based on different values that are considered for an alpha cut, different values are obtained which the table 6 and Fig 1 illustrate it.

Table 6.Sensitivity analysis of cutting-alpha (uncertainty) on the final result

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\mathbf{V}_{\alpha}^{\mathbf{u}}$	29	26	23	18	15	11	13	11	6	3
$\mathbf{V}_{\alpha}^{\mathbf{l}}$	10	9	8	7	6	5	5	4	3	2
$\mathbf{R}_{\alpha}^{\mathbf{u}}$	170	152	134	129	98	85	88	57	43	28
$\mathbf{R}_{\alpha}^{\mathbf{l}}$	156	150	120	107	99	78	79	58	44	26

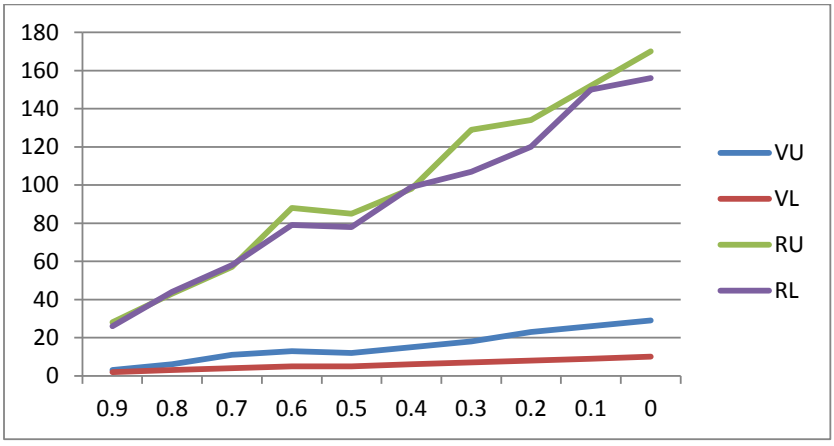

Fig 1. Change in the four main variables in different alpha sections

6. Sensitivity analysis on parameters

Although the above answer is considered as a final answer but if Policy makers in this area are interested that in addition to considering cutting alpha, to have amonts parameters such as the total budget and each budget and also the annual output value, also have the optimum values, for this purpose, the following table represents sensitivity analysis to policymakers:

7. Changes in the total budget

Table 7 and Fig 2 are showing the features of each change only in the alpha 0.5 level.

Table 7.The results of the sensitivity analysis only at the level of alpha = 0.5 for the total budget

The amount in the budget							Variable
140	130	120	110	100	90	80	
21	18	16	15	12	11	5	V_α^u
8	7	7	6	5	2	2	V_α^l
142	125	112	103	85	85	62	R_α^u
134	109	95	91	78	46	17	R_α^l

the following diagrams can be used to analyze the sensitivity of total budget:

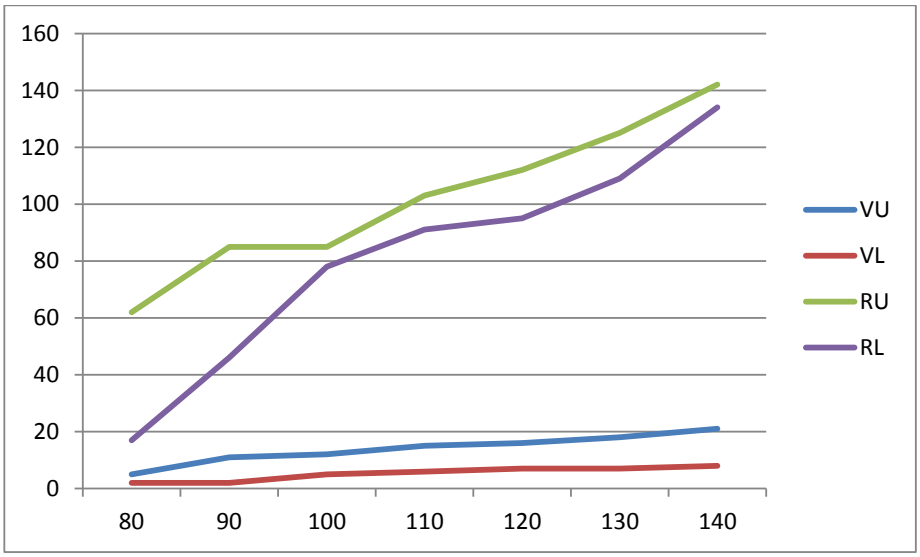

Fig 2. the 4 main variables changing process in the alpha 0.5 level and the whole various budgets

8. Conclusions

This article consider important points in the field of portfolio selection, to be able to challenge the used accurate and transparent data as a basic premise the existing models in the literature. Because of the use of fuzzy data in the present research, to predict the return and utilization of information Financial Statements Return to previous years could cover the Markowitz model.

All previous research in the field of the optimal portfolio selection are conducted by using Neural Network, Goal programming models, Ant colony algorithm, Genetic Algorithm and … are limited to the use of definitive data ,that the model used in this thesis does not have mentioned limitation. Also in conducted research in the portfolio optimization based on fuzzy data, risk or Fuzzy return were related to previous years information, But simultaneously in the this thesis , Information from previous years and predicting the Future years are used in the form of triangular

fuzzy numbers. However Fuzzy numbers are appropriate but the actual number of financial events in previous years should not be neglected and therefore, both types of fragile and Fuzzy data were collected.

In the answer to the main question of this research that "How establishment of Stock portfolio from Mutual funds investment is possible with Fuzzy Approach". It can be stated that the desired fuzzy model in this study, first according to expert opinion has been proposed and extractable. And also based on cutting-alpha can review the terms and analysis sensitivity. Also in answer to the second question will be stated that Shareholders and investors with taking charge the past years actual data of some companies, the survey of the some of the Experts or based on some academic analysis or practical financial markets, extract the numbers and Fuzzy functions and Based on these numbers, review and determine the Stocks status of their choice. The obtained results are in accordance with the basic idea in finance and economics which: whatever the risk an investor is greater, it tends to be more return. For investors who have the emphasis on increased investment returns, obtaining maximum return is considered as the goal and have a higher risk. Whereas the risk getaway investors are seek to minimize the risk so consider the minimizing risk as the goal.

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